A cosmological explanation to the Pioneer anomaly

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Abstract. An earlier paper introduced a new cosmological theory based on the proposition that all four metrical coefficients of space and time change with the cosmological expansion. Such a universal scale expansion would preserve the four-dimensional spacetime geometry and therefore by general relativity most physical relationships. In addition, if the scale expansion were exponential with time, all epochs would be equivalent. The theory resolves several outstanding problems with the standard model based on the Big Bang concept and it better agrees with observations. Four independent observational programs support the SEC theory, which also provides an explanation to the Pioneer anomaly. A possible resolution to the recently discovered discrepancies between optical observations of the planets and their ephemerides is proposed.

Keywords. Pioneer anomaly; space and time expansion; space and time equivalence; scale expansion; space and time symmetry; cosmic drag; tired light redshift; accelerating planets

1. Introduction

The Standard Cosmological Model (SCM) based on the Big Bang has recently come under scrutiny since it has become increasingly clear that the SCM is difficult to reconcile with modern observations, some using the Hubble Space Telescope (HST), see for example Bouwens, Broadhurst and Silk, 1998. The modifications to the SCM demanded by new observational findings are numerous and sometimes mutually contradictory, suggesting that the SCM no longer is an accurate cosmological model.

A basic philosophical and physical difficulty with the SCM is the creation event by which the universe was created instantaneously. This idea is unpalatable because it implies the breakdown of physics at the time of creation, which would make the origin of the universe forever incomprehensible. Identifying an alternative explanation that could address the origin of the universe while staying within the bounds of physical laws would therefore be desirable.

As an alternate approach we could consider various variable transformations of the Friedmann-Robertson-Walker (FRW) line element. One particularly interesting possibility is the simultaneous expansion of all four metrical coefficients rather than just the spatial coefficients. Such a symmetric expansion would be equivalent to scale expansion. Developing this idea leads to the Scale Expanding Cosmos (SEC) theory introduced by the author (Masreliez, 1999).

The structure of the article is as follows: The justification and reasoning that lead to the SEC theory is presented in section 2. Section 3 discusses scale invariance, which is central to the theory. A new phenomenon, cosmic drag, is discussed in section 4. Section 5 discusses tired light redshift and the Pioneer anomaly is introduced in section 6. Section 7 examines the recently reported discrepancies between optical observations of the planets and their ephemerides, showing that this conundrum might be related to the Pioneer anomaly. Observational evidence for the theory is presented in section 8 and section 9 is the summary.

2. Justifying the SEC theory

The celebrated paradox by Parmenides (born 510 BC) poses the following riddle:

> *Only being* ***is*** *- non-being* ***is not****. But, if only being* ***is****, there can be nothing outside this being that articulates it or could bring about change. Hence being must be conceived as eternal,*

uniform and unlimited in space and time.

Clearly, something that exists cannot have been created from nothingness; put differently, *existence rules out non-existence*. Accepting this fundamental conclusion, and taking into account the finding that the universe expands, motivates the search for a cosmological expansion mode without cosmological aging that permits eternal existence. The fact that the universe is scale invariant, as will be discussed in section 3, naturally leads to the SEC theory. Since there is no absolute cosmological reference scale, the cosmological scale of space and time may eternally change with time. The SEC universe evolves by changing all four metrical coefficients of space and time while retaining the relationship between the four metrics. This is equivalent to scale expansion. Changing all four metrical coefficients in Minkowski spacetime by the same factor, i.e. changing the scale of space and time is a well-known gauge symmetry that preserves equivalence. The GR relations are identical for line elements of different scales; all laws of physics modeled by GR are scale invariant.

At first we might reject the idea that the scale of the universe might change with time, but then a valid question would be: "If the scale of objects in the universe were fixed, what could determine this fixed scale?" Since the GR equations are identical regardless of scale all physics should remain the same and no physical process or feature of the universe can determine the scale. Therefore there is no predetermined scale – all scales must be equivalent. If this is the case, it is possible that the cosmological scale is not fixed but may change with time, which immediately suggests that the cosmological expansion could be an expansion of both space and time.

If this were true, there ought to be no physical difference between different epochs; by symmetry reasons all epochs should be equivalent. The scale expansion could well be eternal, which would eliminate the enigmatic creation event. To preserve temporal symmetry the expansion must be a geometrical progression whereby the universe expands by a constant, miniscule, fraction each second. This means that distance and time scales accelerate *relative to a fictional observer in a universe with fixed scale*. In such an exponential scale expansion all locations in space and time would be equivalent.

The SEC line element is (with c=1):

$$ds^2 = e^{2t/T}(dt^2 - dx^2 - dy^2 - dz^2) \tag{2.1}$$

T is the Hubble time (and distance) and t is proportional to atomic time.

This line element is defined relative to a cosmological rest frame generated by the scale expansion as discussed in Masreliez, 1999.

The redshift-distance relation in the SEC is the same as for tired light and may be derived from the geodesic for the line element (2.1), see Masreliez, 1999. It is caused by the scale expansion and is given by the exponential frequency shift of light with time and distance:

$$\nu = \nu_0 \cdot e^{-t/T} = \nu_0 \cdot e^{-d/T}; \qquad (c = 1) \tag{2.2}$$

$$d = T \cdot \ln(\nu_0 / \nu) = T \cdot \ln(z + 1) \tag{2.2a}$$

In the SEC there is also time dilation, see further Appendix A.

3. Scale invariance in the SEC model.

The reader might object that the SEC line element may be transformed into a FRW line element by the transformation *t'=T·exp(t/T)* and that therefore the SEC line element does not offer anything new. *However, the SEC line element is physically equivalent for translations in space and time.*

Obviously, the line element remains the same for spatial translations, for example $\boldsymbol{x}=\boldsymbol{x}'+\boldsymbol{x}_0$ where $\boldsymbol{x}_0$ is a constant position vector.

Temporal translation $t=t'+t_0$ gives:

$$ds^2 = e^{2t_0/T} \cdot e^{2t'/T}(dt^2 - dx^2 - dy^2 - dz^2) \tag{3.1}$$

Einstein's GR equations for this transformed line element are identical to those of the SEC line element; all physical relationships remain the same after a discrete scale change. In general, this also applies to all line elements of the form:

$$ds^2 = S^2 g_{\mu\nu} dx^\mu dx^\nu; \quad S = \text{Scale} \tag{3.2}$$

Thus, scale expansion of flat or curved spacetimes does not alter physical relationships; scaled spacetimes are equivalent and scale invariance is a fundamental, universal, gauge invariance.

The SEC line element models the universe from the perspective of an observer at $t=0$ looking back at the earlier universe for which $t<0$. *By scale invariance the same line element applies to all observers in the SEC regardless of epoch.* Another way to visualize this scale expansion mode would be to allow the increment of proper time to change $ds \Rightarrow ds \cdot exp(t_0/T)$ in (3.1), which would restore the SEC line element (2.1).

Invoking scale invariance takes us "beyond GR" by generalizing it to include discrete scale transformation. This is the main new idea of the SEC theory. In the SEC context GR models the four-dimensional geometry, but *it does not model the progression of time*, which is modeled by the changing scale. It is widely known that GR is a purely geometrical construct that will not model the progression of time. GR does not distinguish between the past and the present. Also, there is no provision for changing the pace of proper time in GR, since proper time corresponds to the global reference increment *ds*. *Yet, it is conceivable that the pace of proper time, as measured out by an atomic clock on a geodesic, might change with the cosmological expansion.*

After the introduction of differential calculus in the 1600s it became common practice to model a continuous process as the limit of stepwise motion with ever shortening intervals. However, this was not obvious to the ancient Greeks; it was challenged by for example Zeno. We should give them credit; it is impossible to visualize truly continuous motion of a rigid body and it is likely that such motion does not exist in Nature. Continuous motion as a limit of arbitrary short intervals in time or space is in conflict with quantum mechanics.

To be able to apply GR for modeling the SEC universe, the pace of proper time must be held constant, for example at the present rate, which permits application of the SEC line element and the GR machinery at this particular epoch. With this approach the universe, *as modelled by GR and the SEC line element*, appears denser in the past and the CMB temperature higher. By the pace of present time the age of the universe equals the Hubble time. *However, this is true for all observers regardless of epoch.* Although fixing the pace of proper time allows us to use GR when modeling the early universe it will give a distorted view that does not a agree with what actually was experienced by an ancient observer, who found the universe exactly as we do today.

Scale expansion checked by discrete, incremental, adjustment in the pace of proper time could cause the metrical coefficients to oscillate relative to an observer in the SEC. This could provide the missing link between GR and Quantum Mechanics (QM). The deBroglie “matter wave” appears as a modulation of the four oscillating spacetime metrical coefficients, the deBroglie/Bohm “pilot function” is the GR geodesic, and the Schrödinger equation may be derived from a GR line element with oscillating metrics (Masreliez, 2004d).

The SEC theory implies new physics, which to some might be a deterrent. However, it does provide a clue to what must be considered the most important aspect of our existence - the progression of time - which in the SEC is modeled by incremental scale expansion. Further properties of the SEC theory are presented in Masreliez (2004a-c) and Masreliez (2005).

The main objective of this paper is to show that the SEC line element accurately models the universe as observed including the Pioneer anomaly, which provides *experimental and repeatable* evidence in support of the theory.

4. Cosmic drag – a new phenomenon.

In the SEC relative velocities of freely moving objects diminish exponentially with a time constant that equals the Hubble time. Also, angular momenta of rotating systems dissipate similarly. This new phenomenon, which follows directly from the GR geodesic for the SEC line element, is derived in Masreliez, 1999, where the following expression for the velocity of a particle freely moving on a geodesic is given:

$$\beta = \frac{\beta_0 \cdot e^{-t/T}}{\sqrt{1-\beta_0^2 + \beta_0^2 \cdot e^{-2t/T}}} \tag{4.1}$$

β is the normalized velocity $\beta = \frac{v}{c}$.

If the velocity initially equals the speed of light so that $\beta_0 = 1$ it follows that $\beta = 1$ for all times. A photon therefore always moves at the speed of light. On the other hand, if the initial velocity is less than the speed of light, it will decrease with time. In particular if $\beta_0 << 1$:

$$\begin{aligned} \beta &= \beta_0 \cdot e^{-t/T} \\ \dot{\beta} &= -\beta / T \end{aligned} \tag{4.2}$$

This is what causes Cosmic Drag in the SEC. Thus, the speed of light remains constant in the SEC, which implies that the Lorentz transformation holds. However, different inertial coordinate systems are no longer equivalent; there is a preferred cosmological reference system, see Masreliez, 1999.

The corresponding expression for angular motion is:

$$r^2 \cdot \dot{\theta}^2 = \frac{r_0^4 \cdot \dot{\theta}_0^{\,2} \cdot (1-\dot{r}^2) \cdot e^{-2t/T}}{r^2 \cdot [1-\dot{r}_0^{\,2} - (r_0 \cdot \dot{\theta}_0)^2] + r^4{}_0 \cdot \dot{\theta}_0^{\,2} \cdot e^{-2t/T}} \tag{4.3}$$

This may be derived from the SEC geodesic by setting φ=0. For velocities much lower than the speed of light we have:

$$r^2 \cdot \dot{\theta} = r_0^{\,2} \cdot \dot{\theta}_0 \cdot e^{-t/T} \tag{4.4}$$

For low velocities the angular momentum decreases exponentially with time-constant *T* in the SEC.

If cosmic drag exists it will have observable consequences, which makes the SEC theory falsifiable. Cosmic drag explains the motion of matter in spiral galaxies and predicts that the planets slowly spiral toward the Sun with accelerating angular velocities. Optical observations in the solar system since the introduction of atomic time have now detected this acceleration as discussed in Masreliez, 1999, and section 7.

5. A few comments in defense of the tired light redshift distance relation.

Currently the belief is widespread that recent supernovae Ia observations definitely refute tired light redshift. This is discussed below and in section 8, where ample evidence in favor of the tired light distance-redshift relation is presented. I will show that cosmological scale expansion, where there is both tired light and time dilation, eliminates several objections.

The most common arguments levelled against tired light are (Wright, 2001):

- ***There is no known interaction that can degrade a photon's energy without also changing its momentum, which leads to a blurring of distant objects. This is not observed.***

In any expanding universe modeled by GR there always is a corresponding relationship between distance and redshift. In the SEC model this redshift-distance relationship is the tired light relation. There is no particular physical "mechanism" creating tired light; in the SEC it is a cosmological spacetime effect. One might say that it is a gravitational effect since it can be derived directly from the GR geodesic, but this would suggest that it is caused by some kind of spatial energy density gradient, which is not the case.

- ***The tired light model can not produce the blackbody spectrum of the CMB.***

It is well known that Planck's spectrum is retained during the cosmological expansion if the energy density is diluted by a factor

$1/(1+z)^4$ and the temperature simultaneously is reduced by a factor $1/(z+1)$, see for example Masreliez, 1999. The Planck spectrum is preserved during cosmological scale expansion in the SEC, which is four-dimensional rather than three-dimensional. According to the SEC line element, all three spatial dimensions expand by the factor *exp(t/T),* or by *(z+1)* according to the redshift relation (2a). Therefore a volume element expands by $(z+1)^3$ and the energy density is diluted by $1/(z+1)^3$. In the SEC the fourth dilution factor comes from the temporal expansion. This new and unfamiliar aspect will here be investigated in some detail.

Consider the scalar product for the momentum:

$$p_\mu p^\mu = m_0{}^2 = g_{\mu\nu} p^\nu p^\mu \tag{5.1}$$

With the SEC line element we get:

$$m_0^2 = e^{2t/T}((p^0)^2 - (\boldsymbol{p})^2) \tag{5.2}$$

The last term is the ordinary spatial momentum vector. For a photon $m_0=0$ and the spatial momentum is equal to the energy:

$$E^2 = (p^0)^2 = \boldsymbol{p}^2 \tag{5.3}$$

p_0 is a constant of motion in GR. Lowering the index we get:

$$E^2 = (e^{-2/T} p_0)^2 = \boldsymbol{p}^2 \tag{5.4}$$

Thus, according to GR the photon energy decreases exponentially with time in the SEC with a time constant *T/2*:

$$|\boldsymbol{p}| = p^0(t) = p_0 \cdot e^{-2t/T} \rightarrow E(t) = \text{Constant} \cdot e^{-2t/T} \tag{5.5}$$

However, in general the momentum satisfies:

$$\boldsymbol{p} = m_0 \frac{d\boldsymbol{x}}{ds} = m_0 \frac{d\boldsymbol{x}}{dt}\frac{dt}{ds} = m_0 \frac{d\boldsymbol{x}}{dt} e^{-t/T}\gamma = m \frac{d\boldsymbol{x}}{dt} e^{-t/T} \tag{5.6}$$

Denoting the momentum *relative to atomic time t* by:

$$\boldsymbol{p_t} = m\frac{d\boldsymbol{x}}{dt}$$

We have: (5.7)

$$\boldsymbol{p} = \boldsymbol{p_t} \cdot e^{-t/T}$$

On the other hand, with the corresponding spatially expanding (de Sitter) line element (with constant temporal metric) we get from (5.1):

$$\begin{aligned} m_0{}^2 &= 0 = (p^0)^2 - e^{2t/T}\boldsymbol{p}^2 = (p_0)^2 - e^{2t/T}\boldsymbol{p}^2 \\ \boldsymbol{p}^2 &= e^{-2t/T}(p_0)^2 \rightarrow E(t) = \text{Constant} \cdot e^{-t/T} \end{aligned} \tag{5.8}$$

Comparing (5.8) to (5.5) the additional factor $e^{-t/T}$ in (5.5) is due to the temporal expansion and provides the fourth dilution factor 1/(z+1). Therefore, Planck's black body spectrum is preserved in the SEC.

Note that GR does not model the universe as experienced by an inhabitant in the past; it models how it would appear if the pace of proper time were constant during the cosmological expansion. As modelled by GR with the SEC line element it appears that the CMB temperature was higher at redshift z with an elevated temperature $Tz=(z+1)T_{CMB}$. In Masreliez, 1999 I reach the same conclusion using the line element (A1.4) of Appendix A. However, by scale invariance, which is not covered by GR, an ancient observer at redshift z saw the same CMB temperature as presently is seen locally. In other words, interpreting the CMB in the context of GR would give the impression that the CMB was generated at an earlier time at redshift z and temperature $(z+1)T_{CMB}$. This is also consistent with black body radiation energy density proportional to $[(z+1)T_{CMB}]^4$, which by the cosmological expansion has been diluted by the factor $1/(1+z)^4$.

The tired light model fails the Tolman surface brightness test.

It agrees with the Tolman test if there also is time dilation (Figure 3). In the SEC all distances remain the same on the average during the cosmological expansion, as measured by timing a light beam, and therefore surface brightnesses decrease in proportion to $1/(z+1)^2$

rather than $1/(1+z)^4$.

The tired light model does not predict the observed time dilation of high redshift supernova light curves.

In the SCM there are two cosmological dimming factors 1/(z+1); one is due to the redshift, the other to time dilation, which often is explained as being caused by a spatial recession velocity. These two dimming factors are also present in the SEC where there is no recession, see Appendix A. Since there is both redshift and time dilation in the SEC, the model agrees with the supernovae observations, see Appendix B and Figure 4.

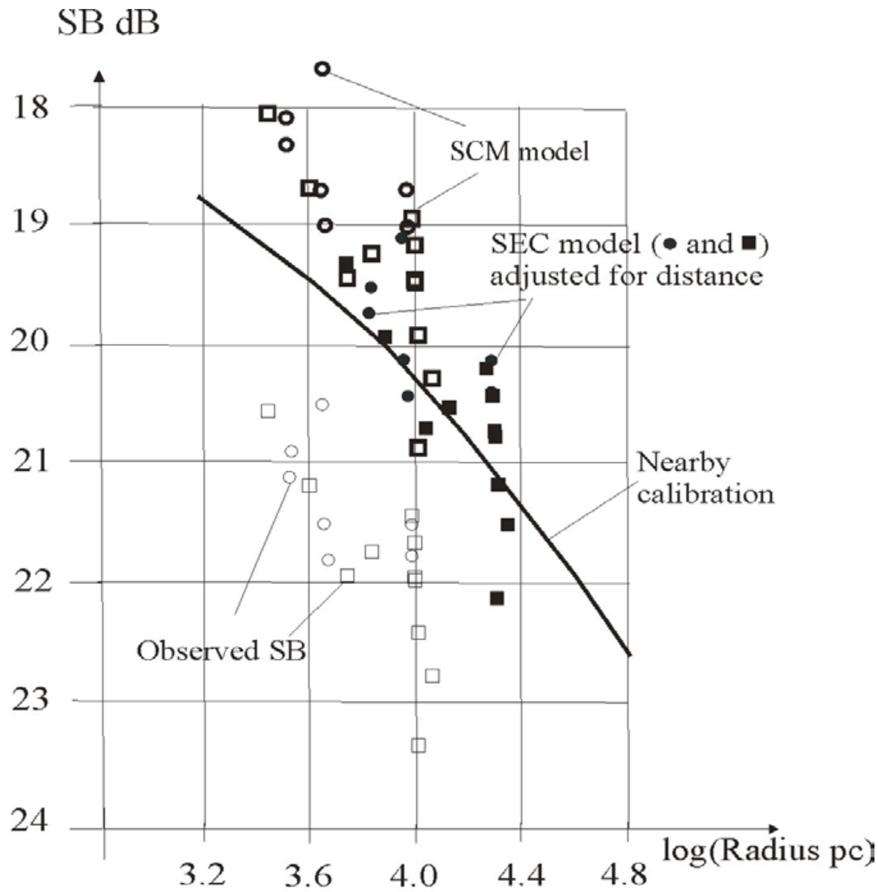

Figure 3: Surface brightness comparison

Circles: I band data for cluster Cl1604+4304 at z=0.90
Squares: I band data for cluster CL1324+3011 at z=0.76

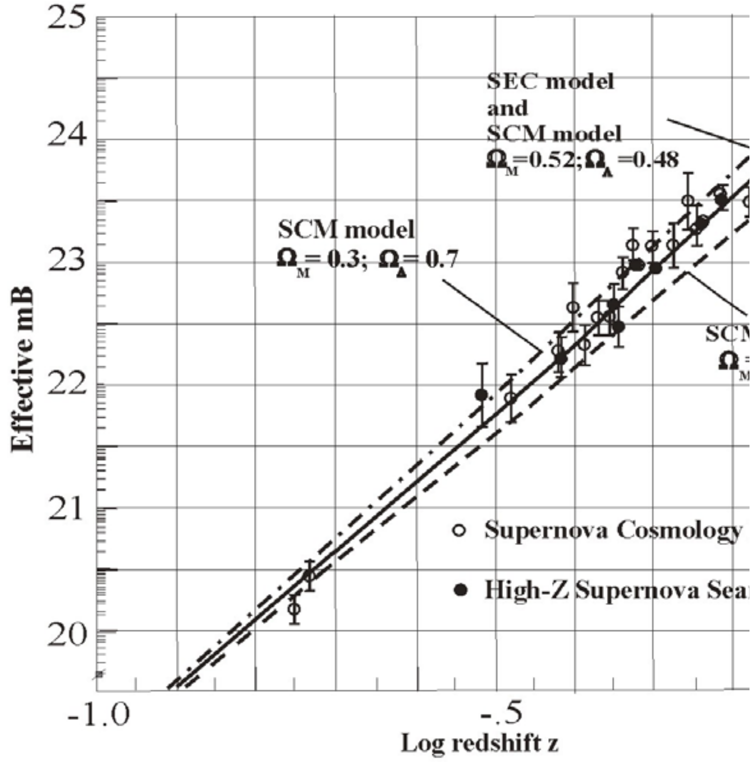

Figure 4: SNe Ia Magnitudes vs. redshift

6. The Pioneer anomaly

6.1 Background.

On March 2, 1972 Pioneer 10 was launched on an Atlas/Centaur rocket from Cape Canaveral. Pioneer 10 was the first space probe sent to the outer planets. After surveying Jupiter on December 4, 1973 it continued outward in the plane of the ecliptic and became our first spacecraft to leave the planetary part of the solar system when it passed beyond the orbit of Pluto in 1983. It was last heard from on January 22, 2003 at a distance of approximately 82 AU from the Sun.

A signal transmitted to Pioneer 10 and returned by the space probe with preserved coherence was monitored up till June 1998. Analysis of the signal at the Jet Propulsion Laboratory (JPL) has yielded new and unique information as reported by Anderson et. al. (2002). A small discrepancy between the measured frequency shift between the uplink and downlink signals and the Doppler shift estimated from ephemeris modeling based on distance ranging became apparent during the 1980s and has remained constant since then. It appears that the space probe is subjected to a tiny, constant acceleration toward the Sun that neither depends on time nor on distance. No physical explanation has yet been found.

This acceleration anomaly is four orders of magnitude larger than the SEC theory's cosmic velocity drag. However, I will suggest that the observed anomaly still might be explained by the SEC theory.

6.2 The Pioneer anomaly is a discrepancy between frequency shifts.

The outward motion of Pioneer 10 is estimated by two independent methods. First, the frequency shift between a 2.11 GHz S-band signal transmitted uplink from the Earth and returned downlink by Pioneer 10 at 2.29 GHz, (where it is compared to a reference signal up-shifted from 2.11 GHz to 2.29 GHz), estimates the velocity of the probe. Second, the motion is estimated based on ranging by measuring the round trip time of phase-modulated pulses returned on the downlink. These ranging estimates are then used as inputs to ephemeris modeling programs at JPL, which estimate the velocity of the probe from which the corresponding Doppler shift is derived (Anderson et. al., 2002).

However, there is an inexplicable discrepancy between the directly measured frequency shift and the modeled Doppler shift estimated from ephemeris tracking. JPL has assumed that the measured frequency shift is a Doppler shift due to the motion of Pioneer 10 and that the discrepancy is caused by an error in the modelled velocity of the probe. If this is the case, the difference might be caused by unmodeled acceleration of unknown origin toward the Sun (in the line of sight) of $(8.74 \pm 1.33) \cdot 10^{-8}$ cm/sec^2 (Anderson et. al., 2002). This acceleration has remained constant for over twenty years, indicating that it does not depend on the velocity or distance. Furthermore, the same constant acceleration has been detected for Pioneer 11 and for the Galileo and Ulysses spacecrafts indicating a new phenomenon possibly of cosmological origin (Anderson et. al., 2002).

Over the past 20 years many different explanations have been suggested, but so far none of them has explained the Pioneer anomaly. Most of these suggestions investigate different mechanisms that might cause the observed small acceleration. However, a few contributors, for example Crawford (1999), have noted that the Pioneer anomaly does not necessarily imply acceleration; the detected discrepancy is between two ways of deriving a certain frequency shift, which this does not necessarily imply a velocity difference. Although JPL implicitly makes the assumption that the directly observed frequency shift is a Doppler shift that accurately corresponds to the velocity of the probe, this may not be the case. If the observed frequency shift is not a pure Doppler shift due to the motion, one cannot conclude that we are dealing with anomalous acceleration.

The relation below is equation (15) in Anderson et. al., 2002. The factor 2 comes from the round trip time, which is twice the distance light-time t_i. $a_p > 0$ is the apparent acceleration anomaly and ν_0 the reference frequency.

$$[\nu_{obs} - \nu_{\mathrm{mod}}]_{DSN} = -\nu_0 \frac{2a_p t_i}{c} \tag{6.1}$$

The JPL definition of this frequency difference is misleading since it is the negative of what one would expect:

$$[\nu_{obs} - \nu_{\mathrm{mod}}]_{DSN} = -[\nu_{obs} - \nu_{\mathrm{mod}}]_{Actual} \tag{6.2}$$

Therefore equation (6.1) should read:

$$[\nu_{obs} - \nu_{\mathrm{mod}}] = \nu_0 \frac{2a_p t_i}{c} \tag{6.3}$$

Contrary to convention the acceleration a_p as defined by JPL is positive in the inward direction. Therefore, the measured frequency ν_{obs} is slightly higher than the modeled frequency ν_{mod} suggesting a blue-shift.

6.3 A cosmological explanation to the Pioneer anomaly

The SEC expansion implies that spacetime is locally curved. However, as shown in Appendix C there exists a local Minkowskian coordinate system in which the SEC theory's cosmic drag disappears and the planetary motions become Newtonian. Since modern ephemerides are constructed by fitting (Post-)Newtonian orbits, they must be based on this locally Minkowskian system. The coordinates of this system as given by (C.2) accelerate relative to the cosmological SEC line element's coordinates. With *t'* the Minkowskian ephemeris time and *t* atomic time of the SEC line element we have from (C.1):

$$\begin{aligned} t' &= T\cosh(r/T)\cdot e^{t/T} \approx (1+\frac{1}{2}(\frac{r}{T})^2)\cdot T\cdot e^{t/T} \approx T\cdot e^{t/T} \\ dt' &\approx e^{t/T}\cdot dt \end{aligned} \tag{6.4}$$

The corresponding radial coordinates are related by:

$$r' = T\sinh(\frac{r}{T})\cdot e^{t/T} \approx r\cdot e^{t/T} \tag{6.5}$$

The Pioneer anomaly may be explained if there is cosmological *scale expansion, and the modeled frequency shift is estimated using locally Minkowskian coordinates.*

With these coordinates the outward radial velocity is:

$$\dot{r}' = \frac{dr'}{dt'} \approx \left(\frac{dr}{dt} e^{t/T} + \frac{r}{T} e^{t/T} \right) \frac{dt}{dt'} \approx \dot{r} + \frac{r}{T} \tag{6.6}$$

The radial coordinate *r'* includes an additional "expanding space" velocity *r/T*. The modeled Doppler shift is in the Minkowskian system:

$$-\nu' \frac{\dot{r}'}{c} = -\frac{\nu'}{c} \left(\dot{r} + \frac{r}{T} \right) = -\nu' \frac{\dot{r}}{c} - \nu' \frac{t_i}{T} \tag{6.7}$$

As above, t_i is the one-way light time and ν′ the modeled frequency based on *t'*. Thus, besides the two-way Doppler shift from the velocity there is extra frequency shift in the Minkowskian system due to a different radial coordinate:

$$\Delta \nu_{\mathrm{mod}\, r} = -\nu_0 \frac{2t_i}{T} \tag{6.8}$$

In this context it is worth mentioning in passing that relation (6.6) implies that the Moon would appear to recede by about 2.8 cm/year due to the choice of Minkowskian coordinates, which is to be compared with the recession rate 3.8 cm/year estimated from laser ranging and ephemeris modeling assuming Newtonian physics.

The modeled downlink frequency is:

$$\nu'_{\text{mod}} = \nu'\left(1 - \frac{2\dot{r}}{c} - \frac{2t_i}{T}\right) \tag{6.9}$$

Besides the frequency contribution (6.8) due to the radial coordinate the use of a Minkowskian temporal coordinate implies that the estimated downlink frequency will be lower than the corresponding frequency based on atomic time. We have:

$$\frac{d\varphi}{dt'} = \frac{d\varphi}{e^{t/T}dt} \qquad \nu_{t'} = \nu_t \cdot e^{-t/T} \tag{6.10}$$

With ν_0 the uplink reference frequency the modeled received downlink frequency becomes expressed in atomic time *t*:

$$\nu'_{\text{mod}} = \nu_0 e^{-2t_i/T}\left(1 - \frac{2\dot{r}}{c} - \frac{2t_i}{T}\right) \approx \nu_0\left(1 - \frac{2\dot{r}}{c} - \frac{4t_i}{T}\right) \tag{6.11}$$

The radial (6.5) and temporal (6.4) transformations together generate a net modeled frequency discrepancy $-4\nu_0 t_i/T$.

On the other hand, the directly observed frequency shift on the down link includes a previously unrecognized redshift due to the SEC theory's tired light distance relation:

$$\nu_{obs} = \nu_0 \cdot e^{-2t_i/T}\left(1-\frac{2\dot{r}}{c}\right) \approx \nu_0\left(1-\frac{2\dot{r}}{c}-\frac{2t_i}{T}\right) \qquad (6.12)$$

The net effect is a frequency discrepancy given by:

$$\nu_{obs} - \nu_{\mathrm{mod}} = \nu_0\left(1-\frac{2\dot{r}}{c}-\frac{2t_i}{T}\right) - \nu_0\left(1-\frac{2\dot{r}}{c}-\frac{4t_i}{T}\right) = \nu_0\frac{2t_i}{T} \qquad (6.13)$$

Setting this expression equal to the right hand of (6.3) yields:

$$a_p = \frac{c}{T} \qquad (6.14)$$

This might explain the Pioneer anomaly. With the above value for the estimated acceleration, $(8.74 \pm 1.33)\cdot 10^{-8}$ cm/s^2, we find that this would imply a Hubble time T in the range 9.9-13.5 billion years, which is in good agreement with estimates of the Hubble time from other sources, for example Tegmark et. al. (2003). A number of people have already noted the coincidence of relation (6.14) and have suggested that the Pioneer anomaly possibly might have cosmological implications.

To further clarify this explanation to the anomaly, consider the following thought experiment. An observer A at r=0, who uses the SEC coordinates, has measured the distance to another observer B, and found it to be constant at $r=r_c$. A transmits this information to B who receives it at t'=0. B uses the Minkowskian coordinates and finds a net velocity relative to A that equals $dr'/dt'=r/T$ since $dr/dt=0$ in relation 6.6. When B receives a second message from A at $t'=2t_i$ with the information that the distance remains constant, B disagrees with A.

It would not be correct to conclude that the use of Minkowskian coordinates is wrong, since GR admits different coordinate representations on equal footing. The planetary orbits may be modeled in Minkowskian spacetime where they will be Newtonian or with the SEC coordinates where they will not be Newtonian. However, if we are dealing with cosmological scale expansion *with atomic time as the temporal parameter* the orbits are no longer Newtonian, since atomic time does not agree with the temporal coordinate of a local Minkowskian system. Had we worked with Minkowskian time instead of atomic time when estimating the observed frequency shift between the uplink and downlink signals, the anomaly would have disappeared. In this case the observed frequency shift would have been almost the same, since the downlink and uplink frequencies would change equally relative to atomic time (the Doppler shift due to the motion merely changes by a factor $2t_i/T$).

However, since we are using Minkowskian coordinates the difference (6.10) no longer applies. As a result there is agreement in (6.13).

Therefore, the Pioneer anomaly might be caused by using different coordinate representations when estimating the observed and modeled frequencies.

This discussion shows that both the Pioneer anomaly and the planetary observational discrepancies may be explained if:

a. Spacetime is curved in the solar system and

b. The modeled frequency is derived with the same, locally Minkowskian, coordinates as is used for the planetary ephemerides.

It also demonstrates how the effect of cosmic drag may be eliminated by using Minkowskian coordinates. This made it impossible to detect deviations from Newtonian orbits before atomic time became available in 1955, which could be the main reason to why the planetary accelerations have not been discovered earlier.

7. Planetary acceleration

In this section I will shed further light on the Pioneer anomaly by suggesting that it might be the "tip of an iceberg"; it might be an important discovery that could lead to a substantial revision of our world-view.

The Pioneer anomaly could be closely related to another observational discrepancy, which, like the Pioneer anomaly, has not yet been explained. This is the recently discovered disagreement between optical observations of planetary positions relative to their computed ephemerides positions. Several authors have independently noted this puzzling problem. (For references see section 7.2).

7.1 Cosmic drag implies planetary acceleration.

The SEC theory's diminishing angular momentum should cause the planets to slowly spiral toward the Sun. It may be shown (Masreliez, 2004c) that Newton's law of gravitation is modified in the SEC and that the gravitational potential is changed by a factor of order $(r/T)^2$:

$$P = \frac{GM}{r} \cdot (1 + O(r/T)^2) \tag{7.1}$$

The term $O(r/T)^2$ is of order 10^{-28} in the solar system. This is negligible, which means that Kepler's third law holds well within observational uncertainties:

$$r^3 \cdot \omega^2 = \text{Constant} \tag{7.2}$$

Combining this law with the cosmic drag angular momentum relation (4.4) gives the planetary accelerations:

Angular acceleration ω: $\frac{d\omega}{dt} = \frac{3\omega}{T} \rightarrow \omega = \omega_0 \cdot e^{3t/T}$ (7.3)

Radial velocity r: $\frac{dr}{dt} = -\frac{2r}{T} \rightarrow r = r_0 \cdot e^{-2t/T}$ (7.4)

Tangential acceleration v: $\frac{dv}{dt} = \frac{v}{T} \rightarrow v = v_0 \cdot e^{t/T}$ (7.5)

Thus, according to the SEC theory the planets spiral toward the Sun with accelerating tangential and angular velocities while their distances from the Sun steadily decrease. The angular (secular) acceleration of the Earth is about 2.8 arcsec/century^2 and the orbital radius currently decreases by about 20 meters per year assuming T=14 billion years.

7.2 Observational evidence for planetary accelerations

Having discovered that the SEC model implies cosmic drag, the question becomes if there exists observational evidence for this phenomenon.

Many pulsars spin down at rates close to the SEC theory's prediction. If a millisecond pulsar were to be slowed down by some other mean, for example friction, it would dissipate heat comparable to the Suns' energy output. The spin-down of pulsars cannot be explained by standard physics but is predicted by the SEC theory (see further Masreliez, 1999).

Regarding the planets, it appears that accelerating angular motions already might have been detected. Recently several independent investigators have reported discrepancies between the optical observations and the planetary ephemerides. The discussions by Yao & Smith (1988, 1991, 1993), Krasinsky et. al. (1993), Standish & Williams (1990), Seidelman et al. (1985, 1986), Seidelman (1992), Kolesnik (1995, 1996) and Poppe et. al. (1999) indicate that residuals of right ascensions of the Sun show a nearly 1″/cy negative linear drift before 1960 and an equivalent positive drift after that date.

A paper by Yuri Kolesnik (Kolesnik, 1996) reports on positive drift of the planets relative to their ephemerides based on optical observations covering thirty years with atomic time. This study uses data from many observatories around the world, and all observatories independently detect the planetary drifts. In personal communication Kolesnik agreed that the noted discrepancies very well might be accelerations and thus quadratic with time.

Recently we have analyzed 240,000 optical planetary observations from year 1750 to 2000. Estimated angular accelerations based on these observations have been published in a joint paper (Kolesnik and Masreliez, 2004).

One might perhaps wonder why these planetary accelerations, have not been detected earlier. In fact, they were discovered a long time ago by several independent investigators, perhaps most prominently Spencer Jones (1939). At the time of the Spencer Jones investigation, time-keeping in astronomy still used Universal Time (UT), which is based on the rotation of the Earth. The detected accelerations may therefore also be explained by a decelerating pace of UT due to decelerating rotation of the Earth (perhaps caused by tidal friction) rather than by accelerating motion of the Earth around the Sun. The rate of deceleration of the Earth's rotation that would account for the observed acceleration of the Sun's motion can be estimated. Correcting UT for this estimated spin-down rate of the Earth and eliminating short-term fluctuations gives "Ephemeris Time (ET)" by which the motion of the Earth and the planets are uniform on the average. However, it also creates an unresolved discrepancy between the spin-down rate of the Earth's rotation and the motion of the Moon, which are related by conservation of angular momentum, (Masreliez, 1999). This problem has been thoroughly investigated by for example Newton (1985) and Dicke (1966) but no good explanation has yet been found.

Today we are facing a curious situation; the drifts detected by optical observations are not apparent when constructing the modern ephemerides. These ephemerides are fitted primarily to radar ranging data between the Earth and the three other inner planets and laser ranging to the Moon. Jet Propulsion Laboratory (JPL) has found that the measured ranges can be fitted excellently to Newtonian ephemerides with relativistic corrections (Post-Newtonian) using *a traditional approach by which the temporal argument implicitly is derived in the ephemeris construction process* (Standish, 1998). This approach was developed at a time when planetary motions were the most reliable time keeper in astronomy, before atomic time became available in 1955. It is commonly believed that this good fit to the ranging data confirms that the planetary orbits are Post-Newtonian with the implicit assumption that the ephemeris coordinate time, ET, is proportional to atomic time, AT.

However, this is not necessarily the case. A good fit does not guarantee that the ephemerides actually are Newtonian in a cosmological reference frame that is not Minkowskian. It is possible that an almost perfect Post-Newtonian fit might be obtained when the ephemeris construction process determines the time base, since this approach automatically might select a local Minkowskian system in which Newtonian orbits apply. If spacetime is curved locally, as is the case with the SEC model, a local Minkowskian system may always be found. But, the temporal coordinate of this local Minkowskian coordinate representation accelerates relative to atomic time, see the Appendix. This would allow perfect ranging data agreement with the Post-Newtonian ephemerides, since the law of gravitation differs by merely an order $(r/T)^2$, which we saw is in the order of 10^{-28}, between the two coordinate representations. In spite of excellent fit to the Post-Newtonian ephemerides, optical observations using atomic time will deviate from the ephemerides, thus explaining the mysterious discrepancy.

Therefore, ranging data cannot *without access to atomic time* verify whether or not Newton's law (with its relativistic corrections) applies in the cosmological reference system. Newtonian ephemerides in a local Minkowskian system might not be Newtonian in a cosmological coordinate system with curved spacetime. Investigating the consequence of this hypothesis, assuming that the SEC theory is correct, we find that the Moon's distance from the Earth changes more slowly than estimated and that the Moon very well could have formed at the same time as the Earth, see section 7.3.

Although modern ephemerides primarily are based on very accurate range measurements to the nearby planets, the ephemerides for the outer planets still use optical observations and Very Long Baseline Interferometry (VLBI). However, the low angular velocities of the outer planets hide their accelerations, which, if detected, easily could be interpreted as being due to observational errors or modeling inadequacies.

Making use of all the available ranging data since the inception of the planetary ranging program some thirty years ago might make it possible to check whether the coordinate time of the ephemerides accelerates relative to atomic time. The temporal acceleration of the ephemeris time-base derived from ranging predicted by the SEC theory is 1/T corresponding to one second quadratic drift relative to atomic time in 30 years. However, the JPL approach of fitting the ephemeris time as closely as possible to a time-base proportional to atomic time would reduce this discrepancy by at least a factor eight making it very difficult to detect. The Earth moves at a speed of 30 km/sec so the ranging discrepancy due to the timing error amounts to about 3-4 km in 30 years. This is comparable to the ranging uncertainties.

In spite of being very small, planetary acceleration could account for the drifts detected by optical observations, since planetary secular accelerations are amplified by a factor three due to the changing radial distance, see relation (7.3) and the Appendix.

The circumstance that the secular planetary accelerations due to cosmic drag are proportional to the motions also explains how they could have been misinterpreted as being caused by a decelerating UT. The semi-acceleration of the Sun (i.e. the Earth's motion in its orbit), deduced by Spencer Jones from solar eclipses, is 1.23 arcsec/cy^2, which suggests that this acceleration primarily could be due to the SEC theory's cosmic drag and not to slowing rotation of the Earth. This could explain the discrepancy between optical observations and the ephemerides and resolve the mismatch between the spin-down of the Earth and the motion of the Moon.

There is at least one study in which the planetary ephemerides are constructed based on AT rather than on a timebase fitted to the observations. This is the investigation by Oesterwinter and Cohen (1972), which concludes that ET based on planetary angular motions drifts relative to AT by about 7 seconds in 50 years. This agrees well with relation (7.3) above, which with T=14 billion years gives a corresponding quadratic temporal drift of 7.5 seconds on 50 years assuming that the drift is caused by a slowing progression of UT.

Also, very early analyses of measured radar ranges by two different teams, one American and one Russian, report positive planetary tangential accelerations based on numerical integrations. Reasenberg & Shapiro (1978) derive positive accelerations of Mercury and Venus based on about 15 years of range measurements. Krasinsky et. al. (1986) also gives positive accelerations derived from radar observations in the interval 1961-1982. These results are consistent with the SEC theory.

Note that the "old ET", which is based on the planetary motions, differs from the temporal argument in the modern ephemerides. The old ET is determined so that the average planetary angular motion is constant relative to the stellar background and therefore corrects for the angular acceleration (7.3). On the other hand, the JPL ephemerides are determined so that the tangential accelerations disappear on the average and primarily corrects for (7.5). These two time bases are not the same and they both differ from AT, which could explain observational inconsistencies.

Summarizing, planetary acceleration as predicted by the SEC theory has recently been detected in several independent studies and will soon be confirmed beyond any reasonable doubt (if they exist) since positional discrepancies increase quadratically with time.

8. Other astronomical observational findings in favor of the SEC theory

Several investigators beginning with Edwin Hubble have argued that astronomical observations better agree with tired light than with the Doppler-like redshift of the SCM. In an important paper Paul LaViolette, 1986 presents clear observational evidence showing that tired light agrees with several cosmological tests without resorting to any of the speculative evolutionary scenarios needed to reconcile the observations with the SCM. But, unfortunately this significant contribution has largely been ignored. Since 1986 our observational capabilities have improved dramatically with new tools like the HST and Very Long Baseline Interferometry (VLBI) and it has become clear that the SCM simply does not agree with the observations.

The following paragraphs will discuss four observational programs, the galaxy number count test, the angular size test, the surface brightness test and the supernovae Ia observations.

8.1 The number count test.

This test was originally designed to discriminate between competing cosmological theories. Any candidate cosmological model should be able to predict how the number of galaxies (galaxy count) increases with distance. Since the luminosity depends on the distance there also is a corresponding test for number count as a function of luminosity. Figure 1 shows a summary from sixteen different number count programs taken from a paper by Metcalf et. al.1995. The SCM model clearly fails the test, while the SEC model agrees well with the observations.

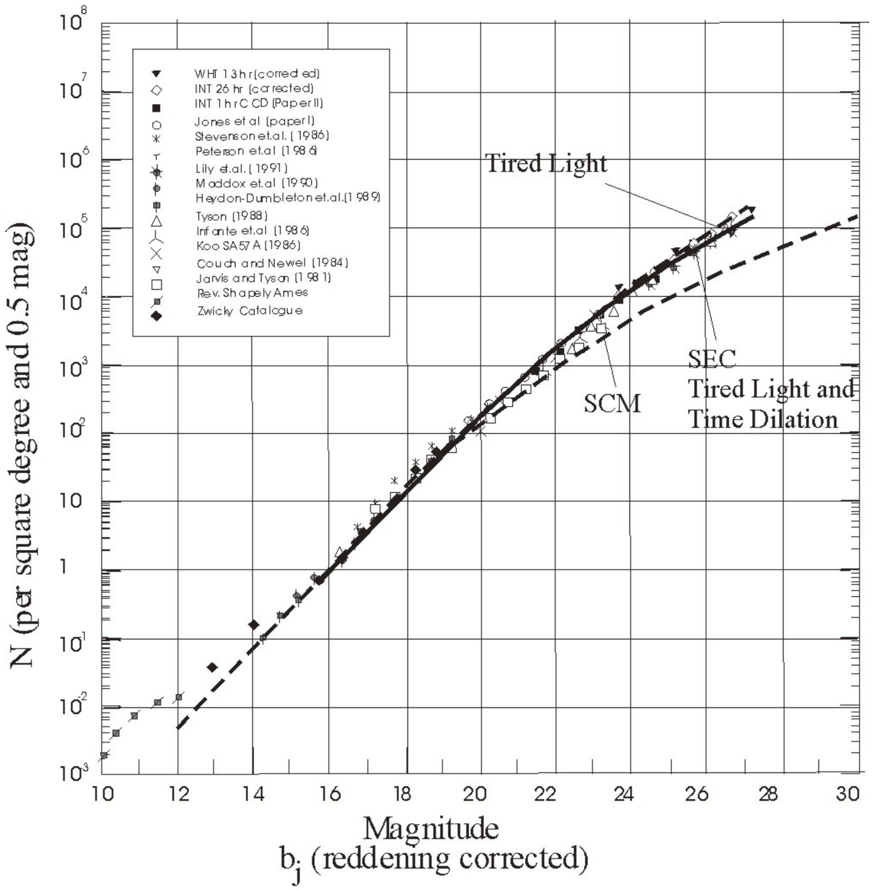

Figure 1: Number count as a function of magnitude.

8.2 The angular size test.

The angular size of a cosmological object, for example a galaxy, may be used to test candidate models. The SCM predicts that the angular size will start to increase with distance beyond a certain distance of minimum size, while the SEC predicts that it will decrease monotonically with increasing distance. Figure 2 is from a paper by Djorgovski and Spinrad, 1981. The tired light prediction has been added. Clearly, The SEC model's agreement with the observations is superior.

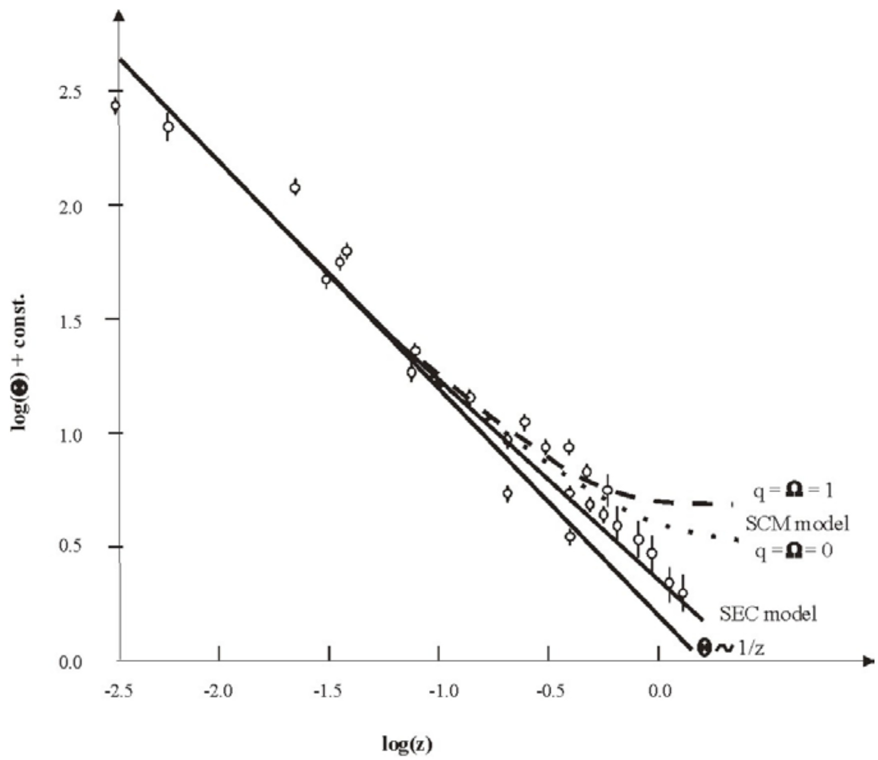

Figure 2: Angular sizes vs. redshift

8.3 The surface brightness test.

The Surface Brightness test is a powerful and robust discriminator between candidate cosmos theories (Tolman, 1930). According the SCM, surface brightness scales with redshift in proportion to $1/(1+z)^4$. One factor $1/(1+z)^2$ is due to redshift and time dilation and an additional factor $1/(1+z)^2$ comes from the viewing angle, which decreases with the cosmological expansion (Lubin and Sandage IV, 2001). On the other hand, in the SEC universe the distance and the viewing angle remain constant during the scale expansion and the surface brightness is proportional to $1/(1+z)^2$, see Appendix A. The difference between the fourth and the second power of (1+z) becomes large at high redshifts, which makes the surface brightness test very powerful. Observational results reported by Lubin and Sandage (2001) show that the SEC theory agrees with observed galaxy surface brightnesses while the SCM does not. The solid line in Figure 3 is the calibrated surface brightness baseline estimated from nearby galaxies. Observed galaxy luminosities in the I-band at z=0.75 and z=0.90 corrected by the factor $(1+z)^2$, and with the radii adjusted to the SEC model, agree well with the local surface brightness (filled symbols). However, there is disagreement with the SCM as shown by the heavier outlined open symbols.

8.4 The supernovae Ia observations.

The recently reported supernovae Ia (SNe Ia) observations by the Supernova Cosmology Project (Perlmutter et. al. 1995) and by the High-Z Supernova Search Team (Schmidt et al. 1998) show that these observations do not agree with the SCM unless the cosmological expansion accelerates. However, as shown in Figure 4 the SNe Ia observations agree well with the theoretical predictions of the SEC model, see further Appendix B. This good agreement with the SEC model is obtained without any adjustable parameters.

Thus, five independent observational programs (including the Pioneer) all agree with theory if there is cosmological redshift and time dilation according to the SEC model. On the other hand, the SCM model disagrees with all five programs.

9. Summary

The Scale Expanding Cosmos theory is based on the proposition that all four metrical coefficients of space and time expand. This corresponds to cosmological scale expansion by which all locations in space and time are equivalent. Scale expansion preserves the spacetime geometry and all laws of physics.

Not only does the SEC resolve a number of conceptual and philosophical problems encountered with the SCM but it also agrees with observations where the SCM fails. In short, the SEC universe looks and behaves just like our universe.

The proposition that the cosmological scale expands is new and perhaps unfamiliar. However, since four-dimensional scale invariance is well-known gauge symmetry it is not unreasonable that the cosmological scale might change with time. Like with the Copernican worldview, which challenged the belief that the Earth is immovable, the SEC theory challenges the belief that the cosmological scale always has remained the same.

The Standard Cosmological Model assumes that the spatial expansion takes effect between galaxies but is not noticeable within them or in the solar system. On the other hand, by the SEC model space and time expands uniformly everywhere and at all levels, which means that spacetime is locally curved.

The Pioneer anomaly may be explained by the inadvertent use of different coordinate representations when estimating the observed and modeled frequencies. Newtonian orbits require that spacetime is Minkowskian, possibly modified by weak gravitational fields (Post-Newtonian). However, if the SEC theory is right the orbits are not Newtonian with the curved cosmological line element. Since modern ephemerides based on ranging are fitted to Newtonian orbits, Minkowskian coordinates are implicitly selected by the ephemeris construction process. But, modeling spacecraft and planetary motions with these coordinates will result in estimates that do not agree with those of observers using atomic time. This might be what causes both the Pioneer anomaly and the optical planetary position discrepancies.

Tired light redshift and time dilation of the SEC theory agrees with several cosmological tests including the number count test, the angular size test, the surface brightness test and it agrees with the supernovae Ia observations without accelerating cosmological expansion.

The fact that the SEC theory agrees well with observational data and resolves many issues makes further investigation worthwhile. Fortunately, cosmic drag will soon either confirm or falsify the theory. Although the planetary accelerations predicted by the theory are quite small, modern astronomical optical observations are sufficiently accurate to detect them, since positional deviations from the Post-Newtonian predictions increase quadratically with time. It might also be possible to confirm the planetary accelerations based on already existing ranging measurements by strictly using atomic time.

The SEC theory is quite unorthodox since it would invalidate basic laws of physics, for example Newton's first law of motion. However, the theory is conceptually simple with only one free parameter, the Hubble time, and it is based on two fundamental symmetries of the universe – scale invariance and equivalence between all locations in space and time.

The reader might still feel somewhat uneasy about the SEC theory because of its far reaching implications and since it relies on new, unproven, physics. However, if one accepts that the scale of spacetime is not absolute but might change with time, all epochs should be equivalent by symmetry. Then it should be possible to model the universe *with the same line element* regardless of epoch, but this is impossible in GR since no continuous variable transformation can replicate the SEC line element with a different temporal argument. We must conclude that either epochs of different scales are not equivalent, or GR falls short when trying to model a scale expanding universe. Generalizing GR to include discrete scale changes permits cosmological, incremental, scale expansion and the SEC theory. Also, this would explain what causes the progression of time and provide the missing link between General Relativity and Quantum Mechanics.

References:

Anderson J. et. al., 2002, "Study of the anomalous acceleration of Pioneer 10 and 11", Phys. *Rev.* ***D65*** *082004*

Bouwens R., Broadhurst T. and Silk J., 1998: *ApJ* ***505****, 556-578*

Bouwens R., Broadhurst T. and Silk J., 1998: *ApJ* ***505****, 579-589*

Crawford D. S., 1999, *astro-ph/9904150*

Dicke R.H.: 1966,"The Secular Acceleration of the Earth's Rotation and Cosmology", *The Earth-Moon System, Plenum Press. pp. 98-164.*

Djorgovski S. and Spinrad H., 1981, *ApJ* ***251****, 417*

Einstein A.: 1917, "Cosmological Considerations on the General Theory of Relativity"*, Trans. from: Sitzungsberichte der Preussischen Akad. d . Wissenschaften*

Hubble E., 1929*, Proc. Nat. Acad, Sci. (USA)* ***15****,168*

Kolesnik Y. B., 1995, *A&A* ***294****, 876*

Kolesnik Y.B., 1996, *in Dynamics, Ephemerides and Astrometry of the Solar System, S. Ferraz-Mello et.al. eds., Reidel, Dordrecht, p.477*

Kolesnik Y, and Masreliez C. J., 2004, "Secular trends in the mean longitudes of the planets derived from optical observations" *AJ.* ***128****, No. 2, p. 878*

Krasinsky G. A., Aleshkina E. Y, Pitijeva E. V., Sveshnikov M. L, 1986*, in Kovalevsky J ans Brumberg V. A. (eds.) Relativety in Celestial Mechanics and Astronomy, p.215*

Krasinsky G. A., Pitjeva E.V., Sveshnikov M.L., Chunayeva L.I.,1993, *Cel. Mec. Dyn. Astron.* ***55****, 1*

LaViolette P.A.: 1986, *ApJl 301, 544-553*

Lubin L. and Sandage A*., 2001, astro-ph/0102213*

Lubin L. and Sandage A., *2001, astro-ph/0102214*

Lubin L. and Sandage A., *2001, astro-ph/0106563*

Lubin L. and Sandage A., *2001, astro-ph/0106566*

Masreliez C.J., 1999, *Ap&SS* ***266****, 399-447*

Masreliez 2004a, "*Scale Expanding Cosmos Theory I-An introduction", Apeiron, July*

Masreliez 2004b, "*Scale Expanding Cosmos Theory II –Cosmic Drag", Apeiron, October*

Masreliez 2004c, "*Scale Expanding Cosmos Theory III –Gravitation", Apeiron, October*

Masreliez 2005, "*Scale Expanding Cosmos Theory IV –A possible link between general relativity and quantum mechanics", Apeiron, January*

Metcalfe N. et. al.: 1995, *M.N.R.A.S. 273, 257-276*

Newton R.R.: 1985, "The Moon's Acceleration and its Physical Origins", *Vol.1&2, John Hopkin's University Press, 1984.*

Oesterwinter C.and Cohen C.J.: *1972, Cel. Mech.* ***5****,317*

Perlmutter, S. et. al., 1995, *ApJ,* ***440,*** *41*

Perlmutter, S, 2003, *Physics Today, April (53-58)*

Poppe P. S. R., Leister N., Laclare F., Delmas C., 1999, *AJ* ***116****, 2574*

Reasenberg R.D, Shapiro I.I.: 1978, *"On the measurement of Cosmological Variations of the Gravitational Constant, ed. L. Halpern (Gainesville:University*

Press of Florida), p.71.

Schmidt B. et.al., 1998, *ApJ, 507, 46*

Seidelman P. K., Santoro E. J., Pulkkinen K. F. 1985, *in Szebenhey V., Balazs B. (eds) Dynamical Astronomy, Austin, Texas, p. 55*

Seidelman P. K., Santoro E. J., Pulkkinen K. F. 1986, *in Kovalevsky J. and Brumberg V. A. (eds.) Relativity in Celestial Mechanics and Astrometry, Kluwer, Dordrecht, p. 99*

Seidelman P. K., 1992, Ferraz-Mello S. et al. *(eds.) Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, Kluwer, Dordrecht, p. 49*

Spencer Jones:1939, *M.N.R.A*.S. ***99****,541*

Standish E. M. 1998, "Time scales in JPL and CfA Ephemerides",

A&A, ***336****, p. 381-384*

Standish E.M., Williams J.G., 1990, *in Lieske J.H. and Abalakin V.K. (eds.) Inertial Coordinate System on the Sky , Kluwer, Dordrecht, p.173*

Tegmark M. et. al., 2003, PRD **69**, 103501

Wright N., 2001*, web page: www.astro.ucla.edu/~wright/tiredlit.htm*

Yao Z-G., Smith C., 1988, in Débarbat S., Eddy J.A., Eichhorn H.K., Upgren A.R. (eds.) *Mapping the Sky, Kluwer, Dordrecht, p.501*

Yao Z-G., Smith C., 1991, *Ap&SS* ***177****, 181*

Yao Z-G., Smith C., 1993, in Muller I.I. and Kolaczek B. (eds.) *Developments in Astrometry and Their Impact on Astrophysics and Geodynamics, Kluwer, Dordrecht, p.403*

APPENDICES

Appendix A. The SEC luminosity relation

In the SCM there is redshift, popularly (but not quite correctly) explained as a Doppler effect. There is also time dilation, which diminishes the photon arrival rate and further reduces the observed flux. The SEC line element may be transformed into a line element of the FRW (Friedmann-Robertson-Walker) type, which models expanding space rather than expanding space and time (scale) and therefore is similar to the SCM line element in that it exhibits both redshift and time dilation. Since all line elements that can be derived via continuous variable transformations are equivalent in General Relativity, the observed luminosity in the SEC universe should diminish not only by the redshift, which contributes with a factor $1/(1+z)$, but there should also be additional cosmological extinction, which contributes by the same factor.

The SEC line element can be transformed into a FRW line element by:

$$u = T \cdot e^{t/T} \quad \text{(A.1)}$$

$$ds^2 = du^2 - (\frac{u}{T})^2 (r^2 d\theta^2 + r^2 \sin^2(\theta)\, d\varphi^2) \quad \text{(A.2)}$$

At $t=0$ and $u=T$ both line elements are Minkowskian and we have $dt=du$. Since these line elements are physically equivalent the photon arrival rate should be the same and since there is both redshift and time dilation in the FRW line element the same is true in the SEC universe.

Another way to see this is to apply the transformation:

$$\begin{aligned} t' &= T \cdot \cosh(r/T) \cdot e^{t/T} \\ r' &= T \cdot \sinh(r/T) \cdot e^{t/T} \end{aligned} \quad \text{(A.3)}$$

The SEC line element transforms into:

$$ds^2 = dt'^2 - dr'^2 - r^2 \cdot e^{2t/T} (d\theta^2 + \sin^2(\theta) \cdot d\varphi^2) \quad \text{(A.4)}$$

Here r and t in the last term are implicitly defined by the two relations above. With this line element there is neither redshift nor time dilation for radial light propagation, but the received light intensity is diluted inversely proportional to the surface element:

$$(r \cdot e^{t/T})^2 = [T \cdot \ln(1+z) \cdot (1+z)]^2$$

This agrees with the SEC apparent luminosity relation (c=1).

$$I = \frac{L}{4\pi \cdot [T \cdot \ln(1+z) \cdot (1+z)]^2} \tag{A.5}$$

However, the luminosity relation presented in Masreliez 1999, which is the traditional relation with tired light redshift, is incorrect since the second factor 1/(1+z) is missing. Again, since there are two dimming factors 1/(1+z) in the SEC model, there is both redshift and time dilation, like in the SCM model.

In the SEC universe the cosmological scale expansion does not change the average distances between galaxies or their angular sizes, since the metrical coefficients of both space and time expand.

Appendix B. The supernovae Ia observations

The SEC distance-redshift relation is given by (2) with c=1:

$$d_{SEC} = T \cdot \ln(z+1) = \frac{1}{H_0} \cdot \ln(z+1) \tag{B.1}$$

H_0 is the Hubble constant. There is an expression by Mattig in flat spacetime for the corresponding distance in the SCM, (Carroll, Press and Turner, 1992):

$$d_{SCM} = \frac{1}{H_0} \int_0^z [(1+x)^2 (1+\Omega_M x) - x(2+x)\Omega_\Lambda]^{-1/2} dx \tag{B.2}$$

where $\Omega_M = \frac{8\pi G}{3H_0^2}\rho_M$ and $\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$ with $\Omega_M + \Omega_\Lambda = 1$.

ρ_M is the (dark) matter density and Λ the cosmological constant.

The apparent luminosity is given by:

$$I_{SCM} = \frac{L}{4\pi \cdot d_{SCM}^2 (1+z)^2} \tag{B.3}$$

For the SEC model the apparent luminosity expression is (see Appendix A):

$$I_{SEC} = \frac{L}{4\pi \cdot d_{SEC}^2 \cdot (1+z)^2} \tag{B.4}$$

These two expressions for the apparent luminosity agree within 0.02 magnitudes in the range $0<z<1$ if Ω_M =0.52 and Ω_Λ=0.48. Furthermore, the SEC luminosity prediction agrees well with the SNe Ia observations as can be seen in Figure 4, which is based on Perlmutter, 2003. This remarkable good agreement with the SEC model is obtained without any adjustable parameters. The dark energy needed to explain the observations in the SCM is implicit with the SEC model.

Appendix C. A locally Minkowskian system

Let's again consider the variable transformation (Masreliez, 1999):

$$t' = T \cdot \cosh(r/T) \cdot e^{t/T}$$

$$r' = T \cdot \sinh(r/T) \cdot e^{t/T} \tag{C.1}$$

The SEC line element is transformed into:

$$ds^2 = dt'^2 - dr'^2 - (r \cdot e^{t/T})^2 \cdot (d\theta^2 + \sin(\theta)^2 \cdot d\varphi^2) \tag{C.2}$$

where r and t are implicitly defined by (C.1). For radial distances within the solar system $r<<T$ it follows from (C.1) that:

$$ds^2 = dt'^2 - dr'^2 - [r' \cdot (1 + O(r/T)^2)]^2 \cdot (d\theta^2 + \sin(\theta)^2 \cdot d\varphi^2) \tag{C.3}$$

The metrical coefficients of the line element (C.3) differ from the Minkowski line element by a fraction $(r/T)^2$, which for the inner planets is of the order 10^{-28}.

It may be shown (Masreliez, 2004c) that gravitational potential in the SEC takes the form:

$$P = \frac{GM}{r}(1 + O(\frac{r}{T})^2) \tag{C.4}$$

For the inner planets there is no observable difference between Minkowskian spacetime and the line element (C.2) since the total residual from the transformation (C.1) and the modified gravitational potential (C.4) is of order $(r/T)^2$. Therefore, fitting the ranging data and time base to Post-Newtonian ephemerides will automatically select line element (C.2) instead of the SEC line element. Perfect fit to Post-Newtonian orbits well within ranging accuracies will obtain, giving the impression that spacetime locally is Minkowskian. However, ephemeris time t' accelerates relative to atomic time t and the optical observations, which measure the planetary positions relative to the stellar background and use atomic time, detect planetary secular acceleration. The radial coordinates r and r' also differ, and although this difference is smaller than the ranging uncertainties they may not be ignored. The diminishing radius contributes $2w/T$ to the secular acceleration of the planets, which is $3w/T$. The remaining balance w/T comes from the tangential acceleration.